# MOTION IN A STRAIGHT LINE

## Speed

Average $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

i.e

$$\text{Average} = \frac{s_1 + s_2 + s_3 \dots \dots}{\left(\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} \dots\right)}$$

**Instantaneous**

$$= \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

## Velocity

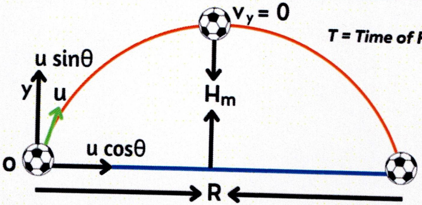
| Average velocity | Instantaneous velocity |
|---|---|
| $\bar{v} = \dfrac{\text{Total displacement}}{\text{Total time taken}} = \dfrac{\Delta x}{\Delta t}$ | $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$ |

## Acceleration

| Average Acceleration | Instantaneous Acceleration |
|---|---|
| $a_{av} = \dfrac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$ | $a_{inst} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$ |

| | |
|---|---|
| **Equation of uniformly Accelerated Motion** | $v = u + at$ |
| (if object is fallen freely under gravity then, value of a is equal to g) | $s = ut + \frac{1}{2}at^2$ |
| | $v^2 - u^2 = 2as$ |

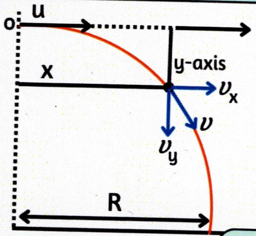
## Projectile Motion




| Time of Flight | Maximum Height | Horizontal Range |
|---|---|---|
| $T = \frac{2u\sin\theta}{g}$ | $H = \frac{u^2\sin^2\theta}{2g}$ | $R = \frac{u^2\sin 2\theta}{g}$ |

| | |
|---|---|
| **Equation of Trajectory**<br>$\theta$=angle of an projectile with horizontal | $y = x\tan\theta\left[1 - \frac{x}{R}\right]$ |

## Horizontal Projectile Motion



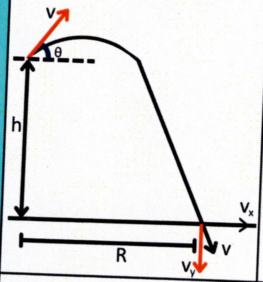

| Time of Flight | Horizontal Range |
|---|---|
| $T = \sqrt{\frac{2H}{g}}$ | $R = u\sqrt{\frac{2H}{g}}$ |

**Velocity of projectile after t**

$$vt = \sqrt{u^2 + (gt)^2}$$

| | |
|---|---|
| **Displacement in $n^{th}$ second** | $S_n = u + \frac{a}{2}(2n-1) = \left(\frac{u+v}{2}\right)t$ |
| **Velocity at any general point** | $v = \sqrt{u^2 + g^2t^2 - 2(u\sin\theta)gt}$ |

**Object projected from some height at some angle $\theta$**

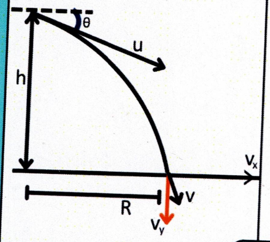



$$t_1 = \frac{T + \sqrt{T^2 + 8\,^h/_g}}{2}$$

where, $T = \frac{2u\sin\theta}{g}$

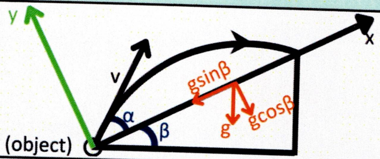
$t_1$=time taken to strike the ground

$$v = \sqrt{u^2 + 2gh}$$




$$t_2 = \frac{\sqrt{T^2 + 8\,^h/_g} - T}{2}$$

## Particles strike inclined plane

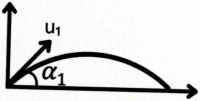



$$T = \frac{2u\sin\alpha}{g\cos\beta} \qquad H = \frac{u^2\sin^2\alpha}{2g\cos\beta}$$

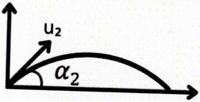
$$R = \frac{2u^2\sin\alpha\cos(\alpha \pm \beta)}{g\cos^2\beta}$$

(+), (-)=particle projected up/down the inclined plane

## Relative Motion b/w two projectiles

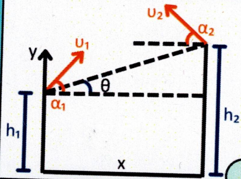



$$u_{12x} = u_1\cos\alpha_1 - u_2\cos\alpha_2$$




$$u_{12y} = u_1\sin\alpha_1 - u_2\sin\alpha_2$$

## Collison of 2 projectile



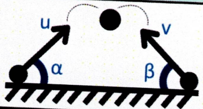

$$\tan\beta = \frac{u_1\sin\alpha_1 - u_2\sin\alpha_2}{u_1\cos\alpha_1 - u_2\cos\alpha_2}$$

$\beta = \text{dir}^n \text{of } v_A(\text{wrt B})\text{making}$

For Collision:

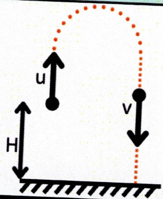
$$\tan\beta = \tan\theta = \frac{h_2 - h_1}{x}$$

## Condition of Collision




$$u\sin\alpha = v\sin\beta$$

## Projection from height

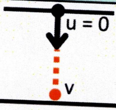



$$H_{mix} = H + \frac{u^2}{2g}$$

$$v = \sqrt{u^2 + 2gH}$$

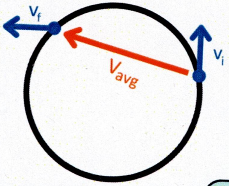
$$T = \frac{\sqrt{u^2 + 2gH} + u}{g}$$

## Free Fall




Height covered in nth second

$$h_n = \frac{g}{2}(2n - 1)$$




Direction of avg. velocity = Along Displacement

Direction of avg. Acceleration = Along $\vec{V_F} - \vec{V_i}$

## Relative Motion

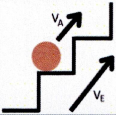

Moving in same direction:

$$\overrightarrow{v_{BA}} = \overrightarrow{v_B} - \overrightarrow{v_A}$$

Moving in opp. direction:

$$\overrightarrow{v_{BA}} = \overrightarrow{v_B} + \overrightarrow{v_A}$$

## Escalator



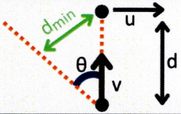

Person moving up:

$$v_{person} = v_A + v_E$$

Person moving down:

$$v_{person} = v_A - v_E$$

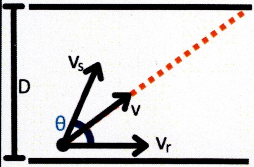
## Minimum Seperation




$$d_{min} = d\sin\theta$$

$$\tan\theta = \frac{u}{v}$$

## River Problem

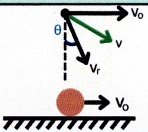



$$T = \frac{D}{v_s \sin\theta}$$

$$\text{Drift} = (v_s\cos\theta + v_r)T$$

$$v = \sqrt{v_0^2 + v_r^2 + 2v_r v_0 \cos\theta}$$

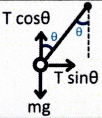
## Rain Problem




v = Actual velocity of a person

$$v = \sqrt{(v_0 v_r \sin\theta)^2 + (v_r \cos\theta)^2}$$

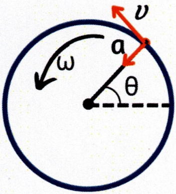
## Pendulum




$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

## Circular Motion




**Angular Displacement**

$$\Delta\theta = \frac{\Delta s}{r}$$

**Angular Acceleration**

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

$$a = r\alpha;\ \alpha = \frac{v^2}{r}$$

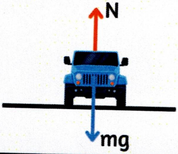
**Angular Velocity**

$$\omega = \frac{\Delta\theta}{\Delta t};\ v = r\omega$$

**Centripetal Force**

$$F = \frac{mv^2}{r} = mr\omega^2$$

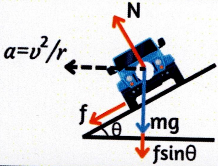
## Motion of Car on a Level Road




$$v^2 \le \mu_s Rg$$

$$v_{max} = \sqrt{\mu_s Rg}$$

## Maximum Speed of the vehicle for a safe turn on the Banked Road




$$v_{max} = \sqrt{\frac{rg(\tan\theta + \mu_s)}{(1 - \mu_s \tan\theta)}}$$